

Kuwait University, Faculty of Science
Dept. of Mathematics & Computer Science
Calculus I Second Mid-Term Test

Time: 75 min.

December 18, 2003

Calculators, Mobile phones, Pagers and all other mobile communication equipments are NOT allowed.

Answer the following questions:

1. [4 points] Use differentials to approximate $\sec^2(44.9^\circ)$.
2. [4 points] Find an equation of the tangent line to the graph of $\sin^3(xy) + \pi = y + x$ at the point whose y -coordinate is 0.
3. [2+3 points]
 - (a) State Rolle's theorem.
 - (b) Let $f(x) = 3x + \frac{9}{5}(x-1)^{\frac{5}{3}} + 1$. Show that there is no $c \in (0, 2)$ such that $f''(c) = 0$. Explain why this result does not contradict Rolle's theorem.
4. [4 points] Find the point on the curve $y^2 = 2x$ that is closest to the point $(1, 4)$.
5. [4×2 points] Let $f(x) = \left(\frac{x}{x+1}\right)^2$.
 - (a) Find the vertical and horizontal asymptotes for the graph of f , if any.
 - (b) Given that $f'(x) = \frac{2x}{(x+1)^3}$. Find the intervals on which f is increasing or decreasing. Find the local extrema, if any.
 - (c) Given that $f''(x) = \frac{2(1-2x)}{(x+1)^4}$. Find the intervals on which the graph of f is concave upward or is concave downward. Find the points of inflection, if any.
 - (d) Sketch the graph of f .

GOOD LUCK

$$1. \quad y = \sec^2 x \quad x = 45^\circ = \frac{\pi}{4} \quad y = \sec^2 \frac{\pi}{4} = 2$$

$$\Delta x = -0.1^\circ = -\frac{\pi}{1800} \quad \Delta y \approx y' \Delta x = 2 \sec^2 x \tan x \Delta x$$

$$\Delta y \approx 2 \sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} \left(-\frac{\pi}{1800}\right) = 2(2)(1)\left(-\frac{\pi}{1800}\right) = -\frac{\pi}{450}$$

$$\sec^2 44.9^\circ \approx 2 - \frac{\pi}{450}$$

$$2. \quad \text{When } y=0, x=\pi \quad 3(xy' + y) \sin^2(xy) \cos(xy) = y' + 1$$

$$\text{at } (\pi, 0), \quad y' = -1 \quad \text{Equation of tangent } y = -(x - \pi)$$

$$3.b. \quad f(x) = 3x + \frac{9}{5}(x-1)^{5/3} + 1 \quad \text{is continuous and differentiable.}$$

$$f'(x) = 3 + 3(x-1)^{2/3}, \quad \text{is continuous on } [0, 2] \text{ but not differentiable}$$

$$\text{at } x=1 \in (0, 2)$$

$$f''(x) = \frac{2}{(x-1)^{1/3}} \neq 0 \quad \text{at any point } c \in (0, 2). \quad \text{This doesn't}$$

contradict Rolle's theorem since $f'(x)$ is not differentiable at $x=1 \in (0, 2)$

4. The distance between (x, y) and $(1, 4)$ is

$$f = \sqrt{(x-1)^2 + (y-4)^2}$$

$$= \sqrt{x^2 - 2x + y^2 - 8y + 17} = \sqrt{\frac{y^4}{4} - 8y + 17}$$

At critical nos,

$$\frac{df}{dy} = 0 = \frac{y^3 - 8}{2\sqrt{\frac{y^4}{4} - 8y + 17}} = 0 \quad \text{when } y=2 \text{ and } x=2$$

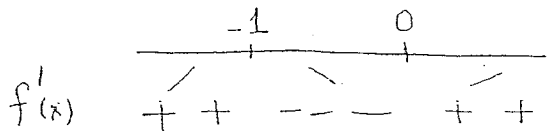
$(2, 2)$ is the closest point to $(1, 4)$

$$5. f(x) = \left(\frac{x}{x+1}\right)^2$$

$\lim_{x \rightarrow \pm \infty} f(x) = 1$; $y = 1$ is a horizontal asymptote.

$\lim_{x \rightarrow (-1)^\pm} \left(\frac{x}{x+1}\right)^2 = +\infty$; $x = -1$ is a vertical asymptote.

$$f'(x) = \frac{2x}{(x+1)^3}$$

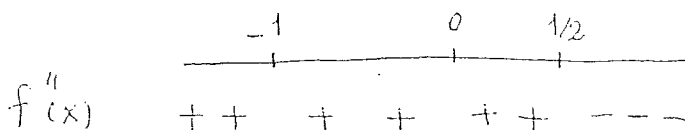


f is increasing on $(-\infty, -1)$ and on $[0, \infty)$

f is decreasing on $(-1, 0]$

f has a local minimum at $(0, 0)$

$$f''(x) = \frac{2(1-2x)}{(x+1)^4}$$



$f(x)$ is concave upward on $(-\infty, -1)$ and on $(-1, \frac{1}{2})$

$f(x)$ is concave downward on $(\frac{1}{2}, \infty)$

f has a point of inflection at $(\frac{1}{2}, \frac{1}{9})$

