Kuwait University, Faculty of Science
Dept. of Mathematics & Computer Science
Calculus I Second Mid-Term Test

Time: 75 min. **December 18, 2003**

Calculators, Mobile phones, Pagers and all other mobile communication equipments are NOT allowed.

Answer the following questions:

1. [4 points] Use differentials to approximate $\sec^2(44.9°)$.

2. [4 points] Find an equation of the tangent line to the graph of
$$\sin^3(xy) + \pi = y + x$$
at the point whose y-coordinate is 0.

3. [2+3 points]

   (a) State Rolle's theorem.

   (b) Let $f(x) = 3x + \frac{9}{5}(x-1)^{\frac{5}{3}} + 1$. Show that there is no $c \in (0,2)$ such that $f''(c) = 0$. Explain why this result does not contradict Rolle's theorem.

4. [4 points] Find the point on the curve $y^2 = 2x$ that is closest to the point $(1,4)$.

5. [4×2 points] Let $f(x) = \left(\frac{x}{x+1}\right)^2$.

   (a) Find the vertical and horizontal asymptotes for the graph of f, if any.

   (b) Given that $f'(x) = \frac{2x}{(x+1)^3}$. Find the intervals on which f is increasing or decreasing. Find the local extrema, if any.

   (c) Given that $f''(x) = \frac{2(1-2x)}{(x+1)^4}$. Find the intervals on which the graph of f is concave upward or is concave downward. Find the points of inflection, if any.

   (d) Sketch the graph of f.

GOOD LUCK

1. $y = \sec^2 x \qquad x = 45^\circ = \frac{\pi}{4} \qquad y = \sec^2 \frac{\pi}{4} = 2$

$\Delta x = -0.1^\circ = -\frac{\pi}{1800} \qquad \Delta y \simeq y' \Delta x = 2 \sec^2 x \tan x \, \Delta x$

$\Delta y \simeq 2 \sec^2 \frac{\pi}{4} \tan \frac{\pi}{4} \left(-\frac{\pi}{1800}\right) = 2(2)(1)\left(-\frac{\pi}{1800}\right) = -\frac{\pi}{450}$

$\sec^2 44.9^\circ \simeq 2 - \frac{\pi}{450}$

2. when $y = 0,\ x = \pi \qquad 3(xy' + y)\sin^2(xy)\cos(xy) = y' + 1$

at $(\pi, 0)$, $y' = -1$ Equation of tangent $y = -(x - \pi)$

3.b. $f(x) = 3x + \frac{9}{5}(x-1)^{5/3} + 1$ is continuous and differentiable.

$f'(x) = 3 + 3(x-1)^{2/3}$, is continuous on $[0,2]$ but not differentiable

at $x = 1 \in (0,2)$.

$f''(x) = \frac{2}{(x-1)^{1/3}} \neq 0$ at any point $c \in (0,2)$. This doesn't contradict Rolle's theorem since $f'(x)$ is not differentiable at $x = 1 \in (0,2)$

4. The distance between (x, y) and $(1,4)$ is

$f = \sqrt{(x-1)^2 + (y-4)^2}$

$= \sqrt{x^2 - 2x + y^2 - 8y + 17} = \sqrt{\frac{y^4}{4} - 8y + 17}$

At critical nos,

$\frac{df}{dy} = 0 = \frac{y^3 - 8}{2\sqrt{\frac{y^4}{4} - 8y + 17}} = 0$ when $y = 2$ and $x = 2$

$(2,2)$ is the closest point to $(1,4)$

5. $f(x) = \left(\dfrac{x}{x+1}\right)^2$

$\lim\limits_{x \to \pm\infty} f(x) = 1$; $y = 1$ is a horizontal asymptote.

$\lim\limits_{x \to (-1)^{\pm}} \left(\dfrac{x}{x+1}\right)^2 = +\infty$; $x = -1$ is a vertical asymptote.

$f'(x) = \dfrac{2x}{(x+1)^3}$

		-1		0	
$f'(x)$	$+ +$		$- - -$		$+ +$

f is increasing on $(-\infty, -1)$ and on $[0, \infty)$

f is decreasing on $(-1, 0]$

f has a local minimum at $(0,0)$

$f''(x) = \dfrac{2(1-2x)}{(x+1)^4}$

		-1		0		$1/2$	
$f''(x)$	$+ +$		$+ \quad +$		$+ +$		$- - -$

$f(x)$ is concave upward on $(-\infty, -1)$ and on $(-1, \frac{1}{2})$

$f(x)$ is concave downward on $(\frac{1}{2}, \infty)$

f has a point of inflection at $(\frac{1}{2}, \frac{1}{9})$